Modern Portfolio Theory has two major conclusions:

- That there is value to **diversification**
- That an asset should be priced according to its risk **relative to the market**

The trade-off between risk and return is one of the central tenets of Finance. If you wish to increase your expected return, you must take on more risk, ceteris paribus.

One thing that dominates the risk/return tradeoff is **diversification**. Diversification is the only "free lunch" in finance: with diversification, one can achieve higher levels of return without exposing themselves to greater risk. This is because of the simple fact that not all asset prices move in tandem with one another—asset prices are not perfectly correlated. Price movements in one asset in a portfolio offset price movements in another asset.

Today, we take the wisdom of diversification for granted. Everyone is given the advice that their portfolio should be diversified. However, there was a time when the common wisdom said the opposite: that people should only invest in a handful of stocks, ones that they closely follow and are familiar with. It was considered unwise to try and pay attention to too many stocks; it was thought that if you were investing in too many different stocks, you were not investing effectively. Diversification was considered foolhardy and wasteful, because it was thought that people only had enough time to research a handful of companies in-depth, and that buying shares of a company you had not researched heavily was foolish.

The famous economics John Maynard Keynes is often quoted for his negative attitude towards diversification, "*It is a mistake to think one limits one's risks by spreading too much between enterprises about one knows little and has no reason for special confidence...*
One's knowledge and experience are definitely limited and there are seldom more than two or three enterprises at any given time in which I personally feel myself entitled to put full confidence.
" and "
A small gamble in a large number of different companies where I have no information to reach a good judgment, as compared with a substantial stake in a company where one's information is adequate, strikes me as a travesty of investment policy.
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The breakthrough on the importance of diversification came with the advent of Modern Portfolio Theory, which was first presented by Harry Markowitz (who received a Nobel Prize in Economics for his achievement).

A portfolio is composed of a group of securities. Each security has its own expected return, risk (which is measured as the standard deviation from expected returns), and weight in the portfolio. There is also a covariance matrix: each security in the portfolio has a certain correlation with each other security in the portfolio.

The insight of modern portfolio theory is that maximum correlation is 1, and a portfolio with 2 securities each with the same return and risk will have less risk than the 2 securities individually. This is because the standard deviation of the portfolio is:

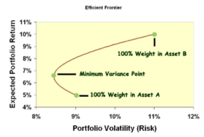
$$\sigma_P = \sqrt{W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2\rho_{AB} W_A W_B \sigma_A \sigma_B}$$

Where:

- σ_P (pronounced "sigma P") is the standard deviation of the portfolio
- ρ_{AB} (pronounced "rho A B") is the covariance between security A and B
- W represents the weight of that security in the portfolio

If $\sigma_A = \sigma_B$, then when the portfolio is not concentrated in a single security—i.e., when none of the W's is equal to 1—the standard deviation decreases than if the portfolio is composed of just a single security. Even if one of the sigmas is less than that of the other, the portfolio sigma can be less than the lower sigma of one of the assets, due to the power of diversification.

Markowitz answered the question, "What is the maximum level of return attainable for a given level of risk?" He found that certain portfolios dominate others—they have higher expected returns for lower risks.



Above is a graph of the **efficient frontier**. This example was constructed using two hypothetical assets, A and B. A has an expected return of 5% and a standard deviation of 9%. B has an expected return of 10% and a standard deviation of 11%. The assets have a covariance of 0.5 with each other.

The expected returns of a portfolio of these assets is simply the weighted average of expected returns, and the standard deviation of each portfolio is computed using the portfolio standard deviation formula above. The minimum variance point represents the lowest attainable level of risk given these two assets. In this example, the minimum variance point represents a weight of about 69.4% in Asset A and 30.6% in Asset B. The expected return from this portfolio is easy to compute: 69.4%*5% + 30.6%*10% = 6.53%. The portfolio standard deviation is the square root of ($69.4\%^2 * .09^2 + 30.6\%^2 * .11^2 + 2*0.5*69.4\%*30.6\%*.09*.11$), which is about .0845.

The area within the curve represents all the portfolios that can be constructed. However, only the points on the upper portion of the line are desirable; points within the curve, or on the lower portion of the line, are undesirable because they offer an inefficient risk-return tradeoff. The points on the line above the minimum variance point are the efficient frontier—the efficient frontier dominates everything below it. In portfolio terms, "dominate" means that a portfolio has a higher expected return than another portfolio, with the same or less level of risk.

The edge below where the line starts to curve shows inadequately diversified portfolios. These portfolios can increase their return for a given level of risk simply by diversifying. Which point on the efficient frontier you want depends on your risk preference, and your individual utility function for returns.

A corollary to modern portfolio theory is the **Two-Fund Theorem**. This theorem states that the efficient frontier can be mimicked using portfolios created with only two securities: a risk-free investment (T-bills) and the theoretical "Global Market Index". Which point on the efficient frontier is achieved depends on how much weight you assign to each of these. The more return you want, the more weight you put into the Global Index—but you increase your risk by doing so.